Information Disclosure in Auctions with Downstream Competition

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July 16, 2017

Abstract

When bidders' valuations are derived from a downstream market in which they may compete, the allocation to the firms with the lowest costs can differ from the allocation that maximizes the ex post valuations of the bidders. I consider the problem of auctioning two goods to bidders whose valuations for a good flexibly depend on their and their rival's costs as well as the identity of the rival. I show that revealing the identities of winners through a sequential auction procedure leads to allocations in which bidders tend to have higher ex post valuations but also higher costs when compared to a simultaneous auction.

Keywords: Auctions; Externalities; Downstream Competition; Multi-Unit

JEL Codes: D43, D44, D47, D82

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1 Introduction

If firms in an auction for licenses vary in terms of how strongly they compete with one another in a post-auction market, their valuations in the auction can depend on the identities of the other winners. This can occur when consumers in the post-auction market incur costs to switch between incompatible products (Klemperer, 1995).

I present a model of such an environment that only places weak restrictions on the post-auction interaction and compare a simultaneous auction for two licenses to a sequential mechanism. Importantly, the latter reveals the first-round winner's identity. I show that the sequential mechanism is more likely to maximize bidder payoffs, while the simultaneous auction always selects the lowest cost bidders. The lowest cost allocation need not maximize the bidders' payoffs due to negative externalities, which if caused by increased competition in the downstream market may be offset by increases in consumer surplus. Intuitively, revealing the prior winner's identity in the sequential format causes the remaining bidders to update their values, leading to a higher likelihood of selecting the bidders with the highest valuations. On the other hand, when bidders update their values they become predictably asymmetric, making it less likely that the lowest cost firms win.

Jehiel and Moldovanu (2006) and the citations therein provide an overview of the consequences of introducing externalities into auction models. This paper is related to a series of papers that analyze a single-good environment in which bidders' payoffs depend on the identity of the winner (Das Varma, 2002a,b; Das Varma and Lopomo, 2010).[1] Das Varma (2002b) finds that revealing the identities of the bidders before an auction increases the auction efficiency (sum of bidder values). Similarly, I find that revealing the first-round winner's identity using a sequential mechanism is more effective at maximizing ex post valuations. On the other hand, Das Varma and Lopomo (2010) studies auctions involving entrants and incumbents, comparing simultaneous to dynamic formats. They find that a simultaneous format may be more efficient because the dynamic format in their model introduces a free riding incentive among incumbents causing strategic non-participation. I provide a different rationale for using a simultaneous format. I show that it is more effective than the sequential format at selecting the bidders with the lowest costs. Both Das Varma and Lopomo (2010) and my paper support the assertion that when bidders are concerned about increased competition in post-auction markets resulting from certain bidders

[1] See Das Varma and Lopomo (2010) for a more discussion of the related literature. Aseff and Chade (2008) study revenue-maximization in a similar setting. Katsenos (2008) considers a similar question in a setting in which there is no distinction between the identities of the bidders.

winning a simultaneous format may be more effective at maximizing social surplus, once downstream consumers are considered.

I next describe the model. The following sections present the results for the simultaneous auction (Section 3) and the sequential mechanism (Section 4). Section 5 concludes. The proofs are in the Appendix.

2 Model

There are $2N$ ($N \geq 2$) bidders competing in an auction for two homogeneous goods. Each demands one unit and has a valuation that depends on the type of the other winner and both winners' cost parameters. Bidders may be one of two types, A and B. There are N bidders of each type.

Let $\pi_S(c_i, c_j)$ denote bidder i's reservation price for a unit of the good if bidder i with cost c_i wins with bidder j with c_j and the bidders are of the same type. Let $\pi_D(c_i, c_j)$ be bidder i's reservation price if bidder j is a different type. I assume that the payoff is zero to losing bidders. The payoff functions, π_S and π_D, are continuous, strictly decreasing in their first argument, and weakly increasing in their second argument.[2] I make the following assumptions for all c_i, c_j:

A1 $\pi_S(c_i, c_j) \leq \pi_D(c_i, c_j)$

A2 $\pi_t(c_i - d, c_j) - \pi_t(c_i, c_j) \geq \pi_t(c_i, c_j) - \pi_t(c_i, c_j - d)$, $t \in \{S, D\}$

Assumption A1 states that an A bidder would prefer to win against an B bidder over an A bidder with the same cost. This assumption is the source of the externality. A2 states that reducing a firm's own cost by d increases the payoff more that increasing that firm's opponent's cost by d. That is, one's own costs are more important than one's rival's.

The private cost parameters, $c_i \in [0,1]$, are distributed independently according to the commonly known distribution, $F(c)$, which is assumed to have a strictly positive density for all $c \in [0,1]$.

The following example shows that these assumptions hold in a market in which consumers have switching costs. It is a slight modification of Example 0 in Klemperer (1995).[3]

[2] Therefore, they are differentiable almost everywhere.

[3] See Farrell and Klemperer (2007) for an overview of this literature.

Example 1. Suppose that firms of type A manufacture products that are compatible with those of other type A firms but incompatible with products made by a type B firm. Assuming n consumers have a reservation price of R for one unit of a good. A fraction σ_A of consumers must pay a switching cost of s to buy from a B firm, while a fraction $\sigma_B = 1 - \sigma_A$ pay s to buy from A. Symmetry requires $\sigma \equiv \sigma_A = \sigma_B$.

If an A and a B firm are in the market with marginal costs c_A and c_B, $s \geq R - c_A > 0$, and $s \geq R - c_B > 0$, then in the unique non-cooperative equilibrium the firms price the goods as if they were monopolists in their respective markets ($p_A = p_B = R$) and earn profits $\sigma n(R - c_A)$ and $\sigma n(R - c_B)$. If there are two As in the market with marginal costs $c_A^1 < c_A^2$, the firms compete for the share of customers in their segment (if $c_A^1 \geq R - s$, the B customers are not served). Under price competition the higher cost firm (firm 2) earns 0 and the lower cost firm (firm 2) earns $\sigma n(c_A^2 - c_A^1)$.

Therefore, for large enough switching costs we always have $\pi_D(c_i, c_j) = \sigma n(R - c_i)$ and $\pi_S(c_i, c_j) = \mathbf{1}\{c_i \leq c_j\}\sigma n(c_j - c_i)$, where $\mathbf{1}\{\cdot\}$ is an indicator function.

3 Simultaneous Vickrey Auction

Consider a sealed-bid auction with the Vickrey pricing rule, where the seller collects bids, awards the objects to the two highest bidders, and charges them the value of the third highest bid. The Vickrey auction rules suggest that it is an equilibrium for the bidders to report their expected value for a good, and it is. However, with externalities the value of the good depends on the expected allocation, and in equilibrium the bidders' expected values have to be consistent with the allocation rule.

There is a symmetric equilibrium in this model, however, because from any bidder's perspective there are $N - 1$ bidders of the same type and N bidders of the other type.[4]

Proposition 1. *In the unique symmetric equilibrium of the simultaneous Vickrey auction firms bid according to*

$$b^{SM}(c) = \int_0^1 ((N-1)\pi_S(c,x) + N\pi_D(c,x))f(x)(1 - F(\max(c,x)))^{2N-2}\,dx \qquad (1)$$

In the proof of this proposition I show that this bid corresponds to the expected valuation of a bidder conditional on winning. Note that the firms submitting the highest bids must be the firms with the lowest costs irrespective of type.

[4] I use symmetric equilibrium to mean that the bids are determined by costs independently of the types.

Although the auction allocates to the firms with the lowest cost, it does not always allocate to the firms with the largest ex post valuation. This is because the firms also care about the identity of the other winner. Suppose that $c_A^1 < c_A^2 < c_B^1$, but

$$\pi_S(c_A^1, c_A^2) + \pi_S(c_A^2, c_A^1) < \pi_D(c_A^1, c_B^1) + \pi_D(c_B^1, c_A^1). \tag{2}$$

Continuity of the payoff functions and the assumptions on the cost distribution imply that this event has positive probability when the inequality in A1 is strict.[5] In other words, the lowest cost firms may win the object without having the highest ex post valuations. However, if the auction allocates to an A and a B firm the allocation must also maximize the firms' ex post valuations. Suppose that $c_A^1 < c_B^1 < c_A^2$, and consider the following.

$$\pi_D(c_A^1, c_B^1) + \pi_D(c_B^1, c_A^1) > \pi_D(c_A^1, c_A^2) + \pi_D(c_A^2, c_A^1) \geq \pi_S(c_A^1, c_A^2) + \pi_S(c_A^2, c_A^1) \tag{3}$$

The first inequality follows from Assumption A2, and the second follows from A1.

4 Sequential Second-Price Auctions

I next consider sequential second-price auctions where the identity and bid of the winner of the first auction is known to those bidding in the second auction. I evaluate second-price auctions for two reasons. The first is tractability. The second-stage environment is asymmetric as the valuations no longer have the same distribution across types, and closed-form solutions to the asymmetric first-price auction are only known in special cases. Second, the second-price auction is the Vickrey auction for a single good, and it compares naturally to the simultaneous Vickrey auction. In the absence of externalities the two formats always produce the same outcome.[6]

I characterize an equilibrium which is symmetric and separating in the first stage. In a Perfect Bayesian Equilibrium, the second-round bids are conditioned on information revealed in the first round, the first-round winner's type and bid.[7] In the second round,

[5]If A1 is strict consider the case where $c_B^1 = c_A^2 + \varepsilon$ for small $\varepsilon > 0$.

[6]If A1 holds with equality, bids are symmetric in both rounds of the sequential procedure (Proposition 2), and hence the sequential auctions and the simultaneous one always select the bidders with the lowest costs.

[7] Revealing more information can cause difficulties with the existence of a separating equilibrium. To see this, suppose that bidders use symmetric, separating strategies in the first round and the type and payment of the winning bidder is revealed. Given symmetry, in a separating equilibrium the payment reveals the cost of the second-highest bidder in equilibrium, and the second-round bidders incorporate this by assuming that

bidders infer the cost of the first-round bidder and have a weakly dominant strategy to bid their inferred value.

To characterize equilibrium bids in the first round it will be necessary to determine the costs at which bidder who bids $\pi_S(c,c')$ ties with a bidder who bids $\pi_D(c'',c')$. Define the mapping $\phi : [0,1]^2 \to [0,1]$ as follows. Given c and c' let $\phi(c,c') \in (0,1)$ be the cost that satisfies $\pi_D(\phi(c,c'),c') = \pi_S(c,c')$, if such a cost exists. Otherwise, set $\phi(c,c') = 1$. Note that for all c and c', $\phi(c,c') \geq c$ and $\phi(c,c')$ is nondecreasing in both arguments. Define $\phi^{-1}(c,c') \equiv \sup\{z|\phi(z,c') \leq c\}$.

In the first round, the bidders' reservation prices are determined by the requirement that at this price they must be indifferent between winning in the first round and abstaining in the first round to continue to the second. In the proof of the next proposition, I show that this reservation price exceeds the expected payment in the second round. This implies that the expected winning bids decrease from the first to second rounds.

Proposition 2. *With sequential second-price auctions, a Perfect Bayesian Equilibrium with symmetric, separating first-round bids must have the following properties. I report the bids of a type A bidder, because the type B bidder's bids are analogous. Bidders who do not win in the first round bid their valuation for the second-round good given the type and cost of the first-round winner. If the first-round winner is a type A (B) with cost c_1, the second-round bid by an A bidder is $\pi_S(c,c_1)$ ($\pi_D(c,c_1)$).*

The first-round bid of a type A bidder is

$$
\begin{aligned}
b_A^{SQ1}(c) = & \frac{N-1}{2N-1}\mathrm{E}[\mathbf{1}\{c_B^1 \leq \phi(c,c)\}\pi_D(c,c_B^1)|c_A^1 = c, c_B^1 \geq c] \\
& + \frac{N}{2N-1}\mathrm{E}[\phi_2(x,c)(\pi_D(c,\phi(x,c)) - \pi_S(c,x))\frac{f_B^1(\phi(x,c))}{f_B^1(c)}|c_A^1 \geq c] \\
& + \frac{N-1}{2N-1}\mathrm{E}[\mathbf{1}\{c_B^1 \geq \phi(c,c)\}\max\{\pi_D(c_B^1,c),\pi_S(c_A^2,c)\}|c_A^1 = c, c_B^1 \geq c] \\
& + \frac{N}{2N-1}\mathrm{E}[\max\{\pi_D(c_A^1,c),\pi_S(c_B^2,c)\}|c_A^1 \geq c, c_B^1 = c].
\end{aligned}
$$

If $b_A^{SQ1}(c)$ is decreasing, these bids form a symmetric, separating equilibrium.

The last two terms in the definition of $b_A^{SQ1}(c)$ have a straightforward interpretation.

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the winning bidder's cost is lower. If the second-highest bidder deviates by pretending to have a lower cost, second-round bidders would infer that the winner has a lower cost and consequently reduce their bids. This gives the losing bidder in the first-round the ability to reduce her expected second-round payment by deviating from a symmetric equilibrium.

These are the expected second-round prices conditional on the event that the bidder ties with the lowest cost type A or type B bidder respectively and would win in the second round. The bidder's first-round bid is not equal to the expected second-round price conditional on tying with the second highest first-round bid and winning in the second round. This is due to the first two terms. The first accounts for the event that $c = c_A^1 < c_B^1 < \phi(c,c)$, in which case the bidder's cost equals that of the lowest cost A opponent, is lower than the lowest cost B opponent, but $\pi_D(c_B^1, c) > \pi_S(c_A^1, c) = \pi_S(c,c)$. The bidder strictly prefers to win in the first round in this event, since he will lose in the second round to the type B bidder. The second term accounts for the first-round winner's influence on the inferences made in the second round about the first-round winner's cost.

I assume for the remainder of the paper that a symmetric, separating equilibrium exists in the sequential mechanism. It is straightforward to verify that this is the case in Example 1.

Example 2 (Example 1 continued)**.** Suppose that $R = s > 1$ and $c \sim U[0,1]$. Then $\pi_D(c,c') = \sigma n(R-c)$, $\pi_S(c,c') = \sigma n(c'-c)\mathbf{1}\{c \leq c'\}$. This implies $\phi(c,c) = 1$ and

$$b_A^{SQ1}(c) = \frac{N-1}{2N-1}\sigma n(R-c) + \frac{N}{2N-1}\mathrm{E}[\sigma n(R-c_A^1)|c_A^1 \geq c],$$

which is decreasing in c. The expression for $b_B^{SQ1}(c)$ is similar. Notice that in this example the allocation is always to an A and a B bidder since any second-round bidder who has the same type as the first-round bidder has a value of zero. This contrasts with the simultaneous auction which might yield outcomes with two winners of the same type.

When $c_A^1 < c_B^1$, in order for a type A bidder to win at all it must win in the first round and in this event the bidder is willing to bid all the way up to $\sigma n(R-c)$. This explains the first term. On the other hand, if $c_A^1 > c_B^1$ a type A bidder may win in either round and is not willing to pay more than their expected second-round price, explaining the second term.

Observe that because the first-round bid functions are symmetric between the types the first good must go to the bidder with the lowest cost. However, once the first good is assigned, the second auction need not allocate to the lowest cost firm. A seller interested in allocating the goods to the lowest cost firms, therefore, prefers the simultaneous format, which prevents the bidders from reacting to the type of the other winner in a second round.

The sequential format always allocates to the bidder with the highest ex post valuation among the second-round bidders. As with the simultaneous format, in the event that the two goods go to bidders of different types, the allocation maximizes ex post valuations.[8]

On the other hand, if the second round allocates to a bidder of the same type, it may be possible to reallocate the goods to increase the bidders' ex post valuations. A second-round bidder of the same type as the first-round winner effectively imposes a negative externality on the first-round winner. However, the next proposition will show that it must be that the sequential auction maximizes the ex post valuations of the bidders more often than the simultaneous auction.

Proposition 3. *If A1 holds strictly, the sequential auction maximizes bidders' ex post values strictly more often than the sealed-bid auction. If A1 holds with equality, the probability of maximizing ex post values is the same in either mechanism.*

5 Conclusion

In the environment I study, a seller interested in allocating to firms with the lowest cost prefers to run a simultaneous Vickrey auction over sequential second-price auctions, while the sequential process allocates to the bidders with the largest ex post valuations more often.

This paper suggests that revealing information about bidders through a sequential auction format harms the ability of the auction to allocate to the bidders with the lowest cost. One criticism of dynamic auction formats is that they may allow for bidders to implement collusive strategies (Klemperer, 2004). Particularly in the switching costs example given in the paper, outcomes of the sequential auction appear collusive since they always yield two firms earning monopoly profits, whereas the simultaneous auction sometimes produces outcomes in which the two firms compete. In other words, there is a sense in which the sequential procedure encourages a form of tacit collusion.

A Appendix

Proof of Proposition 1: Suppose that the firms adopt the strategy described in the statement of the proposition. If the bidders follow this strategy the two highest bids will cor-

[8] Suppose the first good goes to an A firm and the second to a B. Recall that the second good goes to the lowest cost B firm if $c_B^1 \leq \phi(c_A^2, c_A^1)$ where $c_A^1 \leq \phi(c_A^2, c_A^1)$. For the case where $c_A^1 < c_B^1 < c_A^2$, see the inequalities in (3). If $c_A^1 < c_A^2 < c_B^1 < \phi(c_A^2, c_A^1)$, it follows that $\pi_D(c_A^1, c_B^1) > \pi_S(c_A^1, c_A^2)$ and $\pi_D(c_B^1, c_A^1) > \pi_S(c_A^2, c_A^1)$, so that reallocating the goods to the two lowest cost A firms must reduce the ex post valuations. Clearly reallocating to the two lowest cost B firms must also reduce the ex post valuations.

respond to the two lowest draws from $F(c)$. Consider an A bidder who bids according to $b(c')$ but has cost c. The events in which he wins with another A are $c_A^1 \leq c' \leq \min(c_A^2, c_B^1)$ and $c' \leq c_A^1 \leq \min(c_A^2, c_B^1)$. The bidder's ex post valuation in these events is $\pi_S(c, c_A^1)$. His expected valuation is

$$\int_0^{c'} \pi_S(c,x)(N-1)f(x)(1-F(c'))^{2N-2}\,dx + \int_{c'}^1 \pi_S(c,x)(N-1)f(x)(1-F(x))^{2N-2}\,dx,$$

where each integral corresponds to the respective events. Note that there are $N-1$ opposing A bidders, explaining the $N-1$ factor in each integral. These combine to form

$$\int_0^1 \pi_S(c,x)(N-1)f(x)(1-F(\max(x,c')))^{2N-2}\,dx.$$

The argument for the events in which the bidder wins with a type B is analogous. Combining all the terms and setting $c' = c$ gives the expression for $b(c)$. Note that $b(c)$ is decreasing in c. Let $\Pi(c,c')$ be this bidder's expected valuation if he bids $b(c')$ but has cost c and all other bidders use $b(c)$. Since $\Pi(c',c') - b(c') = 0$ and $\Pi(c,c')$ is decreasing in c, $\Pi(c,c') - b(c')$ has the same sign as $c' - c$. This implies that bidding $b(c')$ with $c' \neq c$ when others follow equilibrium strategies is unprofitable.

There cannot be another symmetric equilibrium. Suppose there were and consider the problem of a given A bidder. If the other bidders use a symmetric strategy then by the argument above the bidder's expected value conditional on winning is still $b(c)$, so in any proposed symmetric equilibrium the bidder can improve his payoff by bidding $b(c)$.

Proof of Proposition 2: Optimality of the second-round strategies is obvious, because in the second round the auction corresponds to a second price auction for a single good with independent private values. They are private because their valuations no longer depend on the information or values of other *participating* bidders.

I show how to construct the first-round reservation price, $P(c)$, for bidder i who is an A bidder with cost c (the argument for a B bidder is analogous). Let f_A^1 and f_B^1 be the densities of c_A^1 and c_B^1. Let $f_A^{12}(c_A^1, c_A^2) = (N-1)(N-2)f(c_A^1)f(c_A^2)(1-F(c_A^2))^{N-3}$ be the joint distribution of c_A^1 and c_A^2. Define f_B^{12} analogously. If all other bidders bid according to the reservation price $P(c)$ in the first round and bidder i bids $P(c')$ his payoff is

$$\int_{c'}^{1}\left\{\int_{x}^{\phi(x,c')}\pi_D(c,y)\,dF_B^1(y)+(1-F_B^1(\phi(x,c')))\pi_S(c,x)-(1-F_B^1(x))P(x)\right\}dF_A^1(x)$$
$$+\int_{c'}^{1}\{\pi_D(c,x)-P(x)\}\,(1-F_A^1(x))\,dF_B^1(x)$$
$$+\int_{0}^{c'}(N-1)(1-F_B^1(\max\{\phi(c,x),x\}))(1-F(\max\{c,x\}))^{N-2}\pi_S(c,x)\,dF(x)$$
$$-\int_{0}^{c'}\int_{\max\{c,x\}}^{1}\int_{\max\{\phi(c,x),x\}}^{1}\max\{\pi_D(z,x),\pi_S(y,x)\}\,f_B^1(z)f_A^{12}(x,y)\,dz\,dy\,dx$$
$$+\int_{0}^{c'}N(1-F_A^1(\max\{c,x\}))(1-F(\max\{\phi^{-1}(c,x),x\}))^{N-1}\pi_D(c,x)\,dF(x)$$
$$-\int_{0}^{c'}\int_{\max\{\phi^{-1}(c,x),x\}}^{1}\int_{\max\{c,x\}}^{1}\max\{\pi_D(z,x),\pi_S(y,x)\}\,f_A^1(z)f_B^{12}(x,y)\,dz\,dy\,dx.$$

The first line integrates over events in which the bidder wins in the first round and $c' < c_A^1 < c_B^1$, while in the second line the bidder wins in the first round but $c' < c_B^1 < c_A^1$. It is important to recognize that with positive probability $c' < c_A^1 < c_B^1$ but $\pi_D(c_B^1,c') > \pi_S(c_A^1,c')$. The third and fourth lines integrate over events where $c_A^1 < \min\{c',c_B^1\}$ and bidder i wins in the second round. Similarly, in the last two lines $c_B^1 < \min\{c',c_A^1\}$. After differentiating with respect to c', setting $c'=c$ and then the whole expression to zero, I get

$$P(c)=\frac{N-1}{(2N-1)(1-F(c))^N}\int_{c}^{\phi(c,c)}\pi_D(c,y)\,dF_B^1(y)$$
$$+\frac{N}{(2N-1)(1-F(c))^{N-1}}\int_{c}^{1}(\pi_D(c,\phi(x,c))-\pi_S(c,x))\phi_2(x,c)\frac{f_B^1(\phi(x,c))}{f_B^1(c)}\,dF_A^1(x)$$
$$+\frac{(N-1)(N-2)}{(2N-1)(1-F(c))^{2N-2}}\int_{c}^{1}\int_{\phi(c,c)}^{1}\max\{\pi_D(z,c),\pi_S(y,c)\}\,f_B^1(z)f(y)(1-F(y))^{N-3}\,dz\,dy$$
$$+\frac{N(N-1)}{(2N-1)(1-F(c))^{2N-2}}\int_{c}^{1}\int_{c}^{1}\max\{\pi_D(z,c),\pi_S(y,c)\}\,f_A^1(z)f(y)(1-F(y))^{N-2}\,dz\,dy.$$

which implies that $P(c)=b_A^{SQ1}(c)$ where the latter is defined in the statement of the proposition. This is a type A bidder's reservation price in the first round under the assumption that first-round bidding is symmetric and there is no pooling of types. Setting $b_A^{SQ1}(c)=P(c)$ is a necessary condition for optimality in such an equilibrium.

Similar to the proof of Proposition 1, let $\Pi(c,c')$ be the bidder's expected valuation in the first round if it has a cost of c and bids as if his cost is c' (i.e., bids $b_A^{SQ1}(c')$). The proof that all bidders bidding $b_A^{SQ1}(c)$ forms an equilibrium when $b_A^{SQ1}(c)$ is decreasing is then analogous to the argument in Proposition 1.

Proof of Proposition 3 Let E_{SQ} (E_{SM}, respectively) be the events in which the sequential auction (sealed-bid auction) does not maximize the ex post valuations of the bidders. I show that $E_{SQ} \subseteq E_{SM}$. As discussed above, both auctions maximize the ex post valuations when the goods are allocated to bidders of different types. Thus I restrict attention to realizations that lead to allocations to the same type. I consider cases where the allocation is to two A bidders (the two B bidder case is analogous). E_{SQ} is characterized by the set of inequalities

$$c_A^1 \leq c_A^2 \leq c_B^1 \tag{4}$$

$$\pi_{A,A}(c_A^1, c_A^2) + \pi_{A,A}(c_A^2, c_A^1) < \pi_{A,B}(c_A^1, c_B^1) + \pi_{A,B}(c_B^1, c_A^1) \tag{5}$$

$$\pi_{A,A}(c_A^2, c_A^1) \geq \pi_{A,B}(c_B^1, c_A^1), \tag{6}$$

while E_{SM} is characterized by only (4) and (5). Therefore $E_{SQ} \subseteq E_{SM}$. If A1 holds with equality then (6) is implied by (4) and $E_{SQ} = E_{SM}$.

References

ASEFF, J. AND H. CHADE (2008): "An optimal auction with identity-dependent externalities," *The RAND Journal of Economics*, 39, 731–746.

DAS VARMA, G. (2002a): "Standard Auctions with Identity Dependent Externalities," *Rand Journal of Economics*, 33, 689–708.

——— (2002b): "Who else is bidding? The Pareto optimality of disclosing bidder identities," *Review of Economic Design*, 7, 155–171.

DAS VARMA, G. AND G. LOPOMO (2010): "Non-Cooperative Entry Deterrence in License Auctions: Dynamic versus Sealed Bid," *The Journal of Industrial Economics*, 58, 450–476.

FARRELL, J. AND P. KLEMPERER (2007): "Coordination and Lock-In: Competition with Switching Costs and Network Effects," in *Handbook of Industrial Organization*, ed. by M. Armstrong and R. Porter, Elsevier B.V., vol. 3, chap. 31.

JEHIEL, P. AND B. MOLDOVANU (2006): "Allocative and Information Externalities in Auctions and Related Mechanims," in *Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress, Volume 1*, ed. by R. Blundell, W. K. Newey, and T. Persson, Cambridge University Press, 102135.

KATSENOS, G. (2008): "Simultaneous and Sequential Auctions of Oligopoly Licenses," Unpublished.

KLEMPERER, P. (1995): "Competition when Consumers have Switching Costs: An Overview with Applications to Industrial Organization, Macroeconomics, and International Trade," *Review of Economic Studies*, 62, 515–539.

KLEMPERER, P. D. (2004): *Auctions: Theory and Practice*, Princeton University Press.